

## 2.6: PROBLEM SOLVING AND UNIT CONVERSIONS

### LEARNING OBJECTIVES

- To convert a value reported in one unit to a corresponding value in a different unit using conversion factors.

During your studies of chemistry (and physics also), you will note that mathematical equations are used in many different applications. Many of these equations have a number of different variables with which you will need to work. You should also note that these equations will often require you to use measurements with their units. Algebra skills become very important here!

### CONVERTING BETWEEN UNITS WITH CONVERSION FACTORS

A **conversion factor** is a factor used to convert one unit of measurement into another. A simple conversion factor can convert meters into centimeters, or a more complex one can convert miles per hour into meters per second. Since most calculations require measurements to be in certain units, you will find many uses for conversion factors. Always remember that a conversion factor has to represent a fact; this fact can either be simple or more complex. For instance, you already know that 12 eggs equal 1 dozen. A more complex fact is that the speed of light is  $1.86 \times 10^5$  miles/sec. Either one of these can be used as a conversion factor depending on what type of calculation you are working with (Table 2.6.1).

Table 2.6.1: Conversion Factors from SI units to English Units

English Units	Metric Units	Quantity
1 ounce (oz)	28.35 grams (g)	*mass
1 fluid ounce (oz)	29.6 mL	volume
2.205 pounds (lb)	1 kilogram (kg)	*mass
1 inch (in)	2.54 centimeters (cm)	length
0.6214 miles (mi)	1 kilometer (km)	length
1 quarter (qt)	0.95 liters (L)	volume

\*Pounds and ounces are technically units of force, not mass, but this fact is often ignored by the non-scientific community.

Of course, there are other ratios which are not listed in Table 2.6.1. They may include:

- Ratios embedded in the text of the problem (using words such as *per* or *in each*, or using symbols such as / or %).
- Conversions in the metric system, as covered earlier in this chapter.
- Common knowledge ratios (such as 60 seconds = 1 minute).

If you learned the SI units and prefixes described, then you know that 1 cm is 1/100th of a meter.

$$1 \text{ cm} = \frac{1}{100} \text{ m} = 10^{-2} \text{ m}$$

or

$$100 \text{ cm} = 1 \text{ m}$$

Suppose we divide both sides of the equation by 1m (both the number *and* the unit):

$$\frac{100 \text{ cm}}{1 \text{ m}} = \frac{1 \text{ m}}{1 \text{ m}}$$

As long as we perform the same operation on both sides of the equals sign, the expression remains an equality. Look at the right side of the equation; it now has the same quantity in the numerator (the top) as it has in the denominator (the bottom). Any fraction that has the same quantity in the numerator and the denominator has a value of 1:

$$\frac{100 \text{ cm}}{1 \text{ m}} = \frac{1000 \text{ mm}}{1 \text{ m}} = \frac{1 \times 10^6 \mu\text{m}}{1 \text{ m}} = 1$$

We know that 100 cm is 1 m, so we have the same quantity on the top and the bottom of our fraction, although it is expressed in different units.

### PERFORMING DIMENSIONAL ANALYSIS

Dimensional analysis is amongst the most valuable tools that physical scientists use. Simply put, it is the conversion between an amount in one unit to the corresponding amount in a desired unit using various conversion factors. This is valuable because certain measurements are

more accurate or easier to find than others. The use of units in a calculation to ensure that we obtain the final proper units is called *dimensional analysis*.

Here is a simple example. How many centimeters are there in 3.55 m? Perhaps you can determine the answer in your head. If there are 100 cm in every meter, then 3.55 m equals 355 cm. To solve the problem more formally with a conversion factor, we first write the quantity we are given, 3.55 m. Then we multiply this quantity by a conversion factor, which is the same as multiplying it by 1. We can write 1 as  $\frac{100 \text{ cm}}{1 \text{ m}}$  and multiply:

$$3.55 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}}$$

The 3.55 m can be thought of as a fraction with a 1 in the denominator. Because m, the abbreviation for meters, occurs in both the numerator *and* the denominator of our expression, they cancel out:

$$\frac{3.55 \cancel{\text{m}}}{1} \times \frac{100 \text{ cm}}{1 \cancel{\text{m}}}$$

The final step is to perform the calculation that remains once the units have been canceled:

$$\frac{3.55}{1} \times \frac{100 \text{ cm}}{1} = 355 \text{ cm}$$

In the final answer, we omit the 1 in the denominator. Thus, by a more formal procedure, we find that 3.55 m equals 355 cm. A generalized description of this process is as follows:

$$\text{quantity (in old units)} \times \text{conversion factor} = \text{quantity (in new units)}$$

You may be wondering why we use a seemingly complicated procedure for a straightforward conversion. In later studies, the conversion problems you encounter *will not always be so simple*. If you master the technique of applying conversion factors, you will be able to solve a large variety of problems.

In the previous example, we used the fraction  $\frac{100 \text{ cm}}{1 \text{ m}}$  as a conversion factor. Does the conversion factor  $\frac{1 \text{ m}}{100 \text{ cm}}$  also equal 1? Yes, it does; it has the same quantity in the numerator as in the denominator (except that they are expressed in different units). Why did we not use *that* conversion factor? If we had used the second conversion factor, the original unit would not have canceled, and the result would have been meaningless. Here is what we would have gotten:

$$3.55 \text{ m} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.0355 \frac{\text{m}^2}{\text{cm}}$$

For the answer to be meaningful, we have to *construct the conversion factor in a form that causes the original unit to cancel out*. Figure 2.6.1 shows a concept map for constructing a proper conversion.

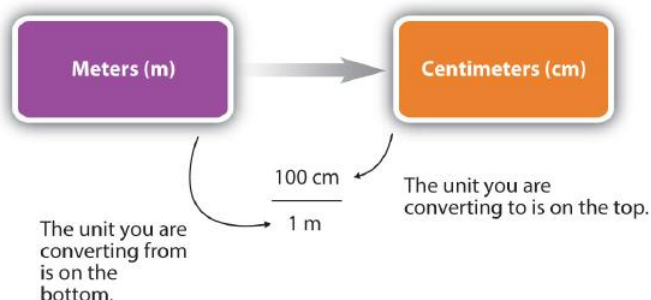


Figure 2.6.1: A Concept Map for Conversions. This is how you construct a conversion factor to convert from one unit to another. Converting meters to centimeters: the unit you are converting is on the bottom of the fraction, and the unit to convert to is on top

## GENERAL STEPS IN PERFORMING DIMENSIONAL ANALYSIS

1. Identify the "**given**" information in the problem. Look for a number with units to start this problem with.
2. What is the problem asking you to "**find**"? In other words, what unit will your answer have?
3. Use **ratios** and conversion factors to cancel out the units that aren't part of your answer, and leave you with units that are part of your answer.
4. When your units cancel out correctly, you are ready to do the **math**. You are multiplying fractions, so you multiply the top numbers and divide by the bottom numbers in the fractions.

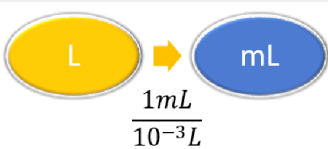
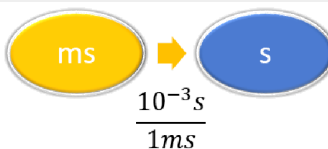
## SIGNIFICANT FIGURES IN CONVERSIONS

How do conversion factors affect the determination of significant figures?

- Numbers in conversion factors based on prefix changes, such as kilograms to grams, are *not* considered in the determination of significant figures in a calculation because the numbers in such conversion factors are exact.
- Exact numbers are defined or counted numbers, not measured numbers, and can be considered as having an infinite number of significant figures. (In other words, 1 kg is exactly 1,000 g, by the definition of kilo-.)
- Counted numbers are also exact. If there are 16 students in a classroom, the number 16 is exact.
- In contrast, conversion factors that come from measurements (such as density, as we will see shortly) or that are approximations have a limited number of significant figures and should be considered in determining the significant figures of the final answer.

### ✓ EXAMPLE 2.6.1

Steps for Problem Solving for Example 2.6.1 and 2.6.2

	Example 2.6.1	Example 2.6.2
<b>Steps for Problem Solving</b>	The average volume of blood in an adult male is 4.7 L. What is this volume in milliliters?	A hummingbird can flap its wings once in 18 ms. How many seconds are in 18 ms?
Identify the "given" information and what the problem is asking you to "find."	Given: 4.7 L Find: mL	Given: 18 ms Find: s
List other known quantities.	$1 \text{ mL} = 10^{-3} \text{ L}$	$1 \text{ ms} = 10^{-3} \text{ s}$
Prepare a concept map and use the proper conversion factor.		
Cancel units and calculate.	$4.7 \text{ L} \times \frac{1 \text{ mL}}{10^{-3} \text{ L}} = 4,700 \text{ mL}$ or $4.7 \text{ L} \times \frac{1,000 \text{ mL}}{1 \text{ L}} = 4,700 \text{ mL}$ or $4.7 \times 10^3 \text{ 2SF, not ambiguous}$	$18 \text{ ms} \times \frac{10^{-3} \text{ s}}{1 \text{ ms}} = 0.018 \text{ s}$ or $18 \text{ ms} \times \frac{1 \text{ s}}{1,000 \text{ ms}} = 0.018 \text{ s}$
Think about your result.	The amount in mL should be 1000 times larger than the given amount in L.	The amount in s should be 1/1000 the given amount in ms.

### ? EXERCISE 2.6.1

Perform each conversion.

- 101,000 ns to seconds
- 32.08 kg to grams
- 1.53 grams to cg

**Answer a:**

$$1.01000 \times 10^{-4} \text{ s}$$

**Answer b:**

$$3.208 \times 10^4 \text{ g}$$

**Answer c:**

$$1.53 \times 10^2 \text{ cg}$$

## SUMMARY

- Conversion factors are used to convert one unit of measurement into another.
- Dimensional analysis (unit conversions) involves the use of conversion factors that will cancel unwanted units and produce the appropriate units.

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